BOOK REVIEW

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Proof and Falsity: A Logical Investigation

By Nils Kürbis
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Nils Kürbis's *Proof and Falsity* takes place against the backdrop of the philosophy of logic developed in Dummett's *The Logical Basis of Metaphysics*. Kürbis introduces Dummett's framework for evaluating the correct logic (Chapters 1 and 2) and shows how, according to it, intuitionistic logic is to be preferred. Kürbis then presents a well-known problem for Dummett's account (Chapter 3) and considers and argues against various putative solutions (Chapters 4–6), before explaining and defending his own preferred approach (Chapter 7). The main conclusion is that, in Kürbis' modified framework, classical (not intuitionistic) logic is justified.

Chapters 1 and 2 provide a thorough introduction to Dummett's proof-theoretic semantics. The different aspects of the view are clearly explained and carefully considered. Together these first two chapters serve as an excellent introduction to proof-theoretic semantics, accessible to anyone with minimal knowledge of logic.

Logic as a discipline strives to provide a precise account of *deductively validity* of argument, that is, of the notion of *following from* in virtue of the meaning of the logical terms (whatever they are). Each account (i.e. each logic) must articulate (the relevant aspects of) the meaning of the logical terms in some way or other and indicate what it is for the conclusion of an argument to follow from the premisses in virtue of these meanings. Proof-theoretic semantics articulates the meaning of logical terms via their characteristic rules of inference. A logic is then given by a calculus, consisting of these rules (perhaps plus further 'structural' rules). An argument is deductively valid in the logic – that is, the conclusion follows from the premisses in virtue of the meaning of the logical terms – just in case the conclusion can be obtained exclusively from the premisses by successive applications of the inference rules; i.e. if there is a proof.

In the natural deduction calculi considered by Dummett, each logical term is governed *only* by rules of introduction and elimination. The introduction rules for a logical term point to the *grounds* for the truth of statements whose main operator is this term, and the rules of elimination point to the *consequences* of the truth of such statements. They are intended as rules of correct assertibility or verifiability; they are intended to be truth-preserving.[1] Dummett's theory of meaning, according to Kürbis, thus has just one primitive notion at its core: truth.

How can a natural deduction calculus be justified as a systematization of the correct logic? How can its account of the meaning of the logical terms be shown to be

1 Correct assertibility and verifiability in logical contexts are tightly connected notions in Dummett's early writings. See, for instance, Dummett (1993a: 165–66) and also Dummett (1993c, 1993d).

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adequate? Presumably, the rules of the calculus would have to be shown to be truth-preserving – i.e. sound. However, this strategy runs into the infamous problem of the justification of deduction. If meaning postulates for the logical terms are to be justified by means of a soundness proof, a vicious circle or an infinite regress ensues. For instance, a justification of rule of Double Negation along these lines will go roughly as follows: if ‘not not *A*’ is a true sentence, then ‘not *A*’ must be untrue, which entails that it’s not the case that *A* itself is untrue; and this, in turn, implies that *A* is true. In neater notation:

$$\mathrm{True}(\neg\neg A) \Rightarrow \neg\mathrm{True}(\neg A) \Rightarrow \neg\neg\mathrm{True}(A) \Rightarrow \mathrm{True}(A).$$

Note that this justification deploys not only negation but also the notion of truth. To be successful, the negation in use must itself satisfy the rule of Double Negation to guarantee the last step of the proof. In other words, the justification presupposes what it purports to justify. Moreover, the proof assumes that a sentence and its negation can’t be simultaneously true (in the move from the first to second step) and also that, if a sentence is not true, then its negation must be (in the move from the second to third step). In short, this justification presupposes the validity of the Law of Non-Contradiction and the Law of Excluded Middle, principles that, given reasonable background assumptions, are equivalent to the very rules whose soundness we seek to establish.

According to Kürbis, Dummett (1993b) claims to break the circle by arguing that inference rules satisfying various conditions don’t need external justification. The analogy is with *stipulative explicit definitions*, which are taken to be true by convention and, thus, self-justifying, due to the fact that they satisfy certain formal requirements (including their conservativeness: stipulative definitions don’t imply any *new* sentences in which the term they purport to define doesn’t occur). In the same vein, Dummett maintains that, if certain formal requirements are met, inference rules articulating the meaning of a given term can be taken to be truth-preserving by convention and, thus, self-justifying.

First, the rules must satisfy a complexity condition: the form of the introduction rules must guarantee that their conclusion is of higher semantic complexity than each of the premisses (cf. Dummett 1993a). This requirement is motivated by Dummett’s meaning-theoretical molecularism, according to which meaning is compositional and so to learn the meaning of a compound expression (i.e. the grounds for its correct assertibility), one must first understand the meaning of its semantically simpler components.

Second, introduction and elimination rules must be *stable* with respect to one another: elimination rules shouldn’t allow us to infer more than what is required for the term to be introduced – i.e. more than what follows directly from the premisses of the introduction rules – and introduction rules shouldn’t demand more for the introduction of the term than what is demanded for the conclusions of the elimination rules (cf. Dummett 1993b).

Rules satisfying those two conditions can be shown to be conservative, just like stipulative definitions (§2.10). Together with a primitive but neutral notion of truth, Dummett maintains, these rules determine the meaning of the logical terms they govern (understood as truth conditions) *completely*. For Kürbis, if this were the case (although Chapter 3 shows that it’s not), it would provide neutral grounds for settling debates about rival logics over the properties of certain logical terms – for instance, whether the

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Law of Excluded Middle and the Law of Non-Contradiction hold – as Dummett claims. Dummettian's proof-theoretic semantics presents itself as an impartial judge for deciding what the correct logic is.

There is, however, something missing in this account, even if inference rules satisfying Dummett's constraints did fully determine the meaning of the logical terms they govern. Unlike vocabulary whose meaning is given by purely stipulative definitions, logical terms come with meanings already attached. Moreover, these pre-existing meanings play a fundamental role in each logic's definition of deductively validity. The rules that purport to articulate the meaning of, e.g., negation need not only to satisfy certain formal constraints, but also to resemble the meaning of our natural language negation, at least to some extent. Otherwise, the calculus would tell us nothing about the validity of arguments in which negations occur. Another way of putting the point is as follows. Given a class of rules satisfying the Dummettian constraints of stability and complexity, and correspondingly defining logical symbols, how is one to know which of those symbols, if any, stands for negation? Again, a bare logical calculus taken on its own remains silent with respect to the validity of arguments in natural language, which is what ultimately matters. This is a pressing (and somewhat underdiscussed) issue for Dummett; it would have been interesting if it had been discussed more by Kürbis.

Deploying his framework, Dummett concludes that the correct logic is not classical but intuitionistic, as some of the classical rules – i.e. for negation – are not stable and violate complexity, whilst intuitionistic connectives satisfy both formal requirements (§2.8, §3.3). However, as Chapter 3 carefully argues, Dummett's framework is seriously flawed. Stability and complexity don't ensure by themselves that the rules of introduction and elimination of a given term *completely* fix the meaning of this term. The intuitionistic rules for *falsum*, in terms of which negation is defined, do not suffice to guarantee the falsity of *falsum* as intended (§3.4). Since intuitionistic negation is defined in terms of *falsum*, its meaning cannot be completely defined by rules of inference either. Dummett was well aware of this issue and proposed ways of solving it, against which Kürbis argues forcefully (§3.5).

Kürbis's diagnosis of the problem is that Dummett's framework favours positive primitives – e.g. truth and assertibility – over negative ones. No negative notions are invoked to define the meaning of the logical terms; no rules of inference that would bring forth falsity or deniability conditions are required. However, inspired by Geach (1972) and some of the post-Dummettian tradition in proof-theoretic semantics, Kürbis maintains that negative primitives are just as important as positive ones for the articulation of meaning (see, for instance, Brandom 2008, Peacocke 2004, Price 1983, Rumfitt 2000 and Tennant 1999).

The three chapters that follow consider the addition of different negative primitives to proof-theoretic semantics in terms of which negation could be defined. Chapter 4 considers the possibility of appealing to a notion of incompatibility as a primitive. Chapter 5 focuses on the idea of taking negation directly as a primitive. Chapter 6 considers taking both assertion and denial as primitives. Each proposal gets its fair share of consideration. The three alternatives are presented in detail and discussed thoroughly and in depth.

Kürbis makes some incisive objections before discarding these views. His arguments are, by and large, convincing, with the exception perhaps of one of the arguments offered against the adoption of primitive notions of assertion and denial along the lines

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of bilateralism (§6.2.4; cf. Price 1983, Rumfitt 2000). Roughly, Kürbis points out that assertion and denial are speech acts, not something inference rules can preserve, or something that can be assumed and discharged later. However, as Kürbis himself notes, some bilateralists – e.g. Rumfitt – often talk about (correct) assertibility and deniability rather than assertion and denial: whilst the latter are speech acts, the former aren't. The preservation of correct assertibility and deniability makes perfect sense, as does making (and discharging) the assumption that a proposition is assertible. What's more, in the Dummettian framework, correct assertibility and correct deniability are intimately connected with truth and falsity. Thus, even though known implementations of bilaterialism might be inferior to Kürbis's own proposal, from a Dummettian perspective there seems to be a rather small conceptual gap between both approaches, if at all.

In the final chapter of the book, Kürbis outlines his own proposal, i.e. to incorporate falsity as a primitive to proof-theoretic semantics, alongside truth. As with truth, no substantial assumptions are to be made regarding falsity. As in Dummett's framework, logical terms must be governed by stable rules of introduction and elimination satisfying the complexity requirement. But unlike in Dummett's framework, logical terms can be characterized not just in terms of rules that preserve *truth*, but also in terms of rules that preserve *falsity*.

This is an elegant and well-rounded proposal that suits the spirit of Dummett's framework very well. It is also nicely executed. Kürbis provides a calculus implementing his idea, consisting of stable operational rules for the logical terms for which complexity holds, together with two structural rules that specify how truth- and falsity-preserving rules may be combined. The operational rules, Kürbis argues, suffice to define the meaning of each logical term, fixing the flaw in the Dummettian approach.

Moreover, the structural rules are shown to be eliminable. This entails a normalization result for the system and, hence, the subformula property. As a consequence, the inference rules are conservative, as required by Dummett's original account. The resulting logic, however, is not intuitionistic but classical. Kürbis thus offers a novel justification of classical logic on Dummettian grounds.

Each view considered in the book is presented with the utmost clarity. Technical notions are introduced in a timely manner and formal results are proved in a reader-friendly manner. Moreover, some chapters can be skipped without much loss of the thread, which makes the book excellent teaching material in an upper-level undergraduate or graduate course. For its clarity of exposition and the elegance and simplicity of its core proposal, I highly recommend *Proof and Falsity* to anyone with an interest in proof-theoretic semantics or, more generally, in logical consequence.

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